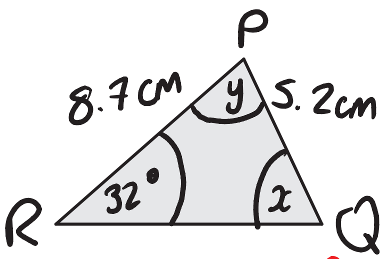
1. In triangle RPQ,

$RP = 8.7$ cm
$PQ = 5.2$ cm
Angle $PRQ = 32°$

(a) Assuming that angle PQR is an acute angle,
calculate the area of triangle RPQ.
Give your answer correct to 3 significant figures.

angle < 90°



Sine rule:
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

① work out angle Q

$$\frac{\sin 32}{5.2} = \frac{\sin x}{8.7} \quad ①$$

$$\sin x = 8.7 \times \frac{\sin 32}{5.2}$$

$$x = \sin^{-1}\left[8.7 \times \frac{\sin 32}{5.2}\right] \quad ①$$

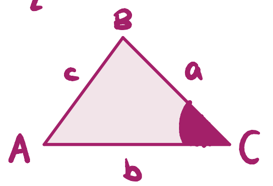
$$x = 62.44853188°$$

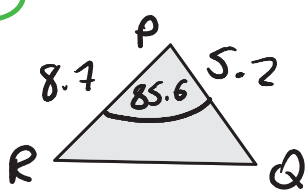
② work out angle P

$$y = 180 - 32 - 62.448...$$
$$= 85.55146812°$$

③ calculate area ①

Area of triangle = $\frac{1}{2}ab\sin(C)$





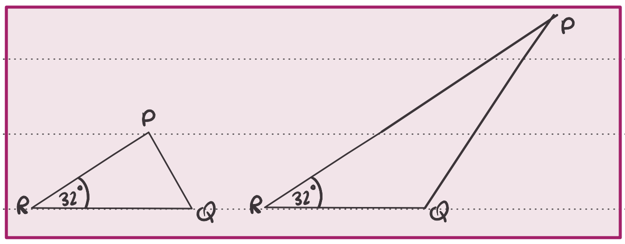
Don't round when you type into calculator

$$\text{Area} = \tfrac{1}{2} \times 5.2 \times 8.7 \times \sin(85.6°)$$
$$= 22.55185522$$
$$= 22.6 \ (3sf)$$

22.6 ① cm²

(4)

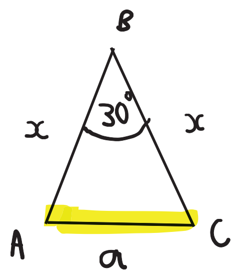
(b) If you did not know that angle PQR is an acute angle, what effect would this have on your calculation of the area of triangle RPQ?

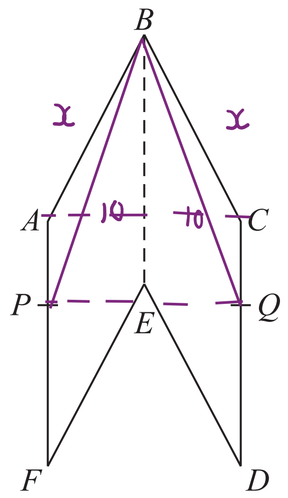


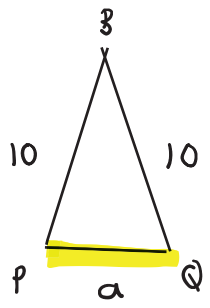
We would need to work out the area of two triangles ① (one when Q is acute and one when obtuse) since areas could differ

(1)

2. The diagram shows a hexagon *ABCDEF*.







$a^2 = b^2 + c^2 - 2bc\cos A$.

Two congruent shapes are identical in size and shape.

ABEF and *CBED* are congruent parallelograms where $AB = BC = x$ cm.
P is the point on *AF* and *Q* is the point on *CD* such that $BP = BQ = 10$ cm.

Given that angle $ABC = 30°$,

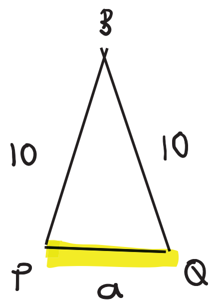
prove that $\cos PBQ = 1 - \frac{(2-\sqrt{3})}{200}x^2$ in terms of x

ABC: $a = a$. $b = x$. $c = x$. $A = 30°$ $\cos 30 = \frac{\sqrt{3}}{2}$. (1)

$a^2 = x^2 + x^2 - (2 \times x \times x \times \cos 30)$

$a^2 = 2x^2 - 2x^2\left(\frac{\sqrt{3}}{2}\right)$. $a^2 = 2x^2 - \frac{\cancel{2}x^2\sqrt{3}}{\cancel{2}}$. $a^2 = 2x^2 - x^2\sqrt{3}$.

$a^2 = x^2(2-\sqrt{3})$. (1)



$a^2 = b^2 + c^2 - 2bc\cos A$.

$a = a$. $b = 10$. $c = 10$. $A = \angle PBQ$

(1)

$a^2 = 10^2 + 10^2 - (2 \times 10 \times 10 \times \cos PBQ)$.

$x^2(2-\sqrt{3}) = 200 - 200\cos PBQ$ (1)

$200\cos PBQ + x^2(2-\sqrt{3}) = 200$. $200\cos PBQ = 200 - x^2(2-\sqrt{3})$.

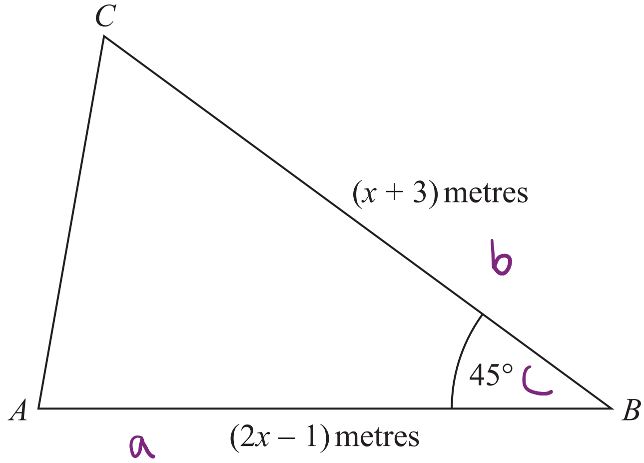
$\cos PBQ = \frac{200 - x^2(2-\sqrt{3})}{200}$.

(Total for Question is 5 marks)

$\cos PBQ = \frac{\cancel{200}}{\cancel{200}} - \frac{x^2(2-\sqrt{3})}{200}$.

$\boxed{\cos PBQ = 1 - \frac{(2-\sqrt{3})}{200}x^2.}$ (1)

3.



The area of triangle ABC is $6\sqrt{2}$ m^2.

$A = \frac{1}{2} ab\sin C$.

Calculate the value of x.
Give your answer correct to 3 significant figures.

$$6\sqrt{2} = \frac{1}{2}(2x-1)(x+3) \times (\sin 45) \quad (1)$$

$$6\sqrt{2} = \frac{1}{2}(2x^2+5x-3) \times \frac{\sqrt{2}}{2} \quad (1)$$

$$6\sqrt{2} = (2x^2+5x-3) \times \frac{\sqrt{2}}{4}$$

$$\div\frac{\sqrt{2}}{4}\left(\quad 24 = 2x^2+5x-3 \quad\right)\div\frac{\sqrt{2}}{4}$$

$$2x^2+5x-3 = 24 \quad (1)$$

$$2x^2+5x-27=0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad (1)$$

$$a = 2. \quad b = 5. \quad c = -27$$

$x = 2.63104...$

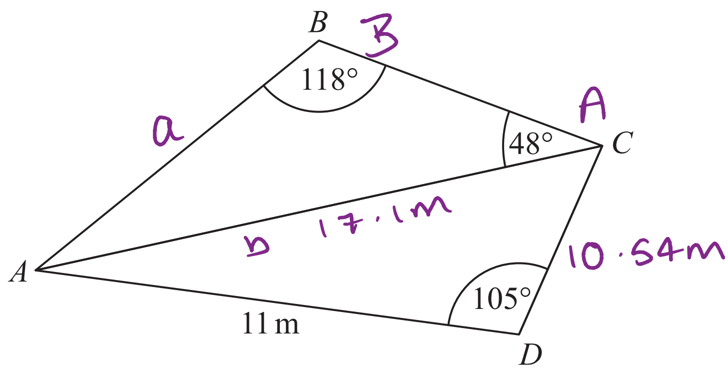
$x = -5.131043...$

(1)

$x = 2.63.$ ....

(Total for Question is 5 marks)

4. ABC and ADC are triangles.



The area of triangle ADC is $56\,\text{m}^2$

Work out the length of AB.
Give your answer correct to 1 decimal place.

Area of triangle $= \frac{1}{2}ab\sin C$

$56 = \frac{1}{2} \times 11 \times CD \times \sin 105°$

$56 = 5.313 \times CD$

$(\div 5.313) \quad (\div 5.313)$

$10.54 = CD$

$a^2 = b^2 + c^2 - 2bc\cos A$

$AC^2 = 11^2 + 10.54^2 - 2 \times 11 \times 10.54 \times \cos 105°$

$AC^2 = 292.107$

$\sqrt{} \quad \sqrt{}$

$AC = 17.1\text{m}$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{AB}{\sin 48°} = \frac{17.1}{\sin 118°}$

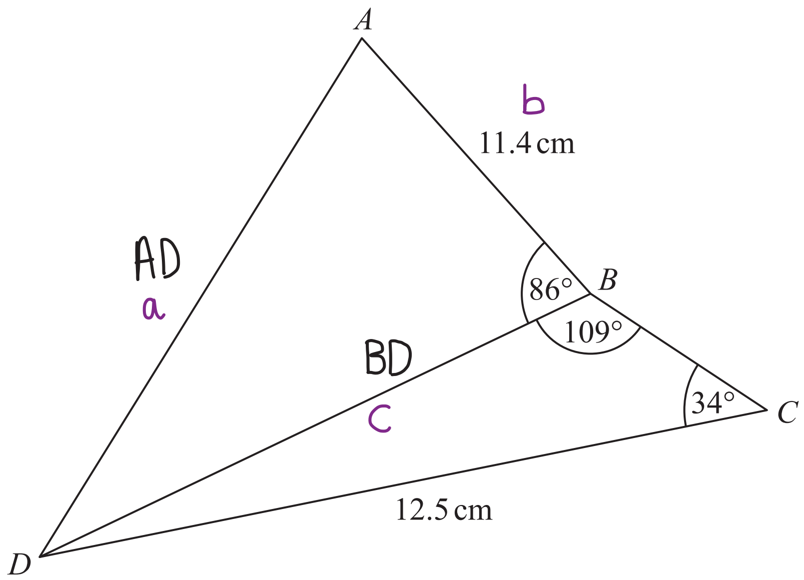
$AB = \sin 48° \left(\frac{17.1}{\sin 118°}\right)$

$AB = 14.4\text{m}$ (1.d.p)

.......14.4....... m

(Total for Question is 5 marks)

5.



Work out the length of *AD*.
Give your answer correct to 3 significant figures.

Finding BD from triangle BCD:

2 angles, 2 sides

Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{BD}{\sin(34)} = \frac{12.5}{\sin(109)}$ (1)

$BD = \frac{12.5}{\sin(109)} \times \sin(34)$

$BD = 7.39\ldots$ (1)

(use the exact value in the rest of the question)

Finding AD from triangle ABD:

3 sides, 1 angle

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos(A)$

$(AD)^2 = (AB)^2 + (BD)^2 - (2 \times (AB) \times (BD) \times \cos(86))$

$(AD)^2 = \left(11.4^2 + (7.39\ldots)^2\right) - (2 \times 11.4 \times 7.39\ldots \times \cos(86))$ (1)

$(AD)^2 = 172.85\ldots$ (1)

$AD = \sqrt{172.85}$

$AD = 13.147$

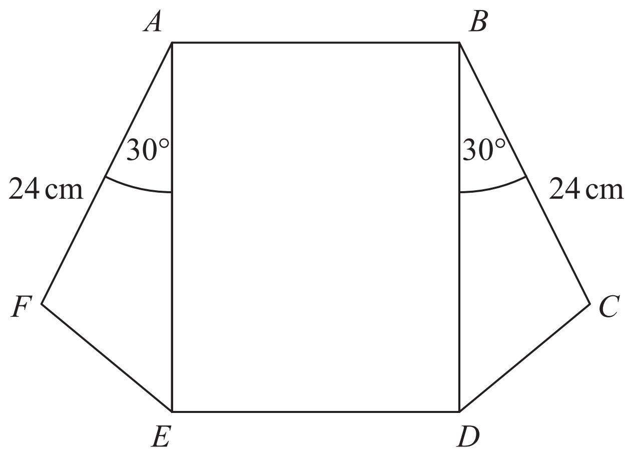
$AD = 13.1$ to 3SF

(1)

13.1 .... cm

(Total for Question is 5 marks)

6. The diagram shows a rectangle, $ABDE$, and two congruent triangles, AFE and BCD.



area of rectangle $ABDE$ = area of triangle AFE + area of triangle BCD

$$AB : AE = 1 : 3$$

Work out the length of AE.

Area of $\triangle AFE$ = Area of $\triangle BCD$ = $\frac{1}{2}ab\sin C$.

$\frac{1}{2}ab\sin C = (\frac{1}{2})(24)(AE)(\sin 30) = 6AE$. (1)

$\therefore$ Area of $\square$ ABDE = $6AE + 6AE = 12AE$. (1)

Let $AB = x$ and $AE = 3x$. (1)

Area of $\square$ ABDE $= (x)(3x) = 3x^2$

Area of $\square$ ABDE $= 12AE = 12(3x) = 36x$ (1)

$\therefore 3\cancel{x}^2 = 36\cancel{x}$.

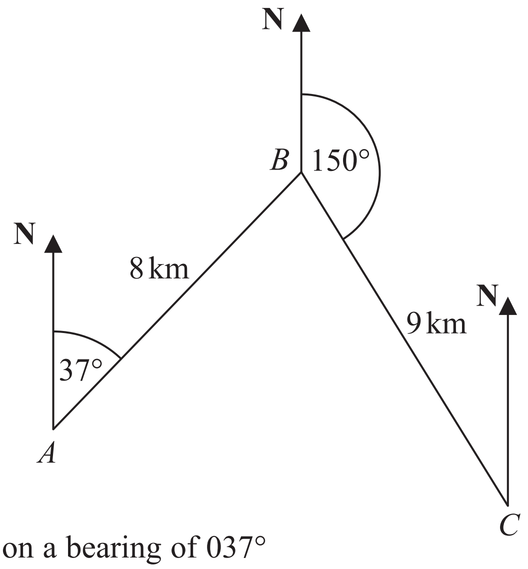
$3x = 36$

$AE = 3x = \underline{\underline{36 \text{ cm}}}$.

36 .... cm

(Total for Question is 4 marks)

7. The diagram shows the positions of three towns, Acton (A), Barston (B) and Chorlton (C).

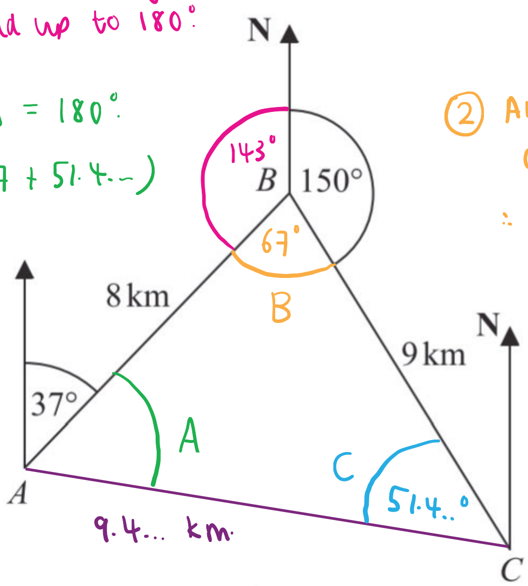


Barston is 8 km from Acton on a bearing of 037°
Chorlton is 9 km from Barston on a bearing of 150°

Find the bearing of Chorlton from Acton.
Give your answer correct to 1 decimal place.
You must show all your working.

Bearing = 37 + A°

① co-interior angles add up to 180°



⑤ Angles in Δ = 180°

$A = 180 - (67 + 51.4...)$

$= 61.5786...°$

(1)

② Angles around a point add up to 360°. (1)

$\therefore B = 360 - (150 + 143) = 67°$

④ Sine rule: $\frac{\sin C}{8} = \frac{\sin 67}{9.41..}$

$(9.41...) \sin C = 8 (\sin 67)$

(1) $\sin C = 0.78175...$

$\therefore C = 51.42131479...°$

③ Find length AC using cosine rule:

$a^2 = b^2 + c^2 - 2bc\cos A$

$AC^2 = 8^2 + 9^2 - (2 \times 8 \times 9 \times (\cos 67))$

$AC^2 = 88.7347175...$ (1)

$\therefore AC = 9.41991069\,5..$ km

⑥ Bearing $= 37 + A$

$= 37 + 61.5786...$

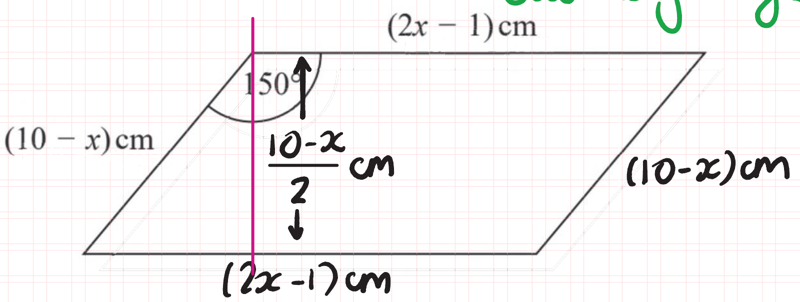
$= \underline{\underline{098.6°}}$ (1d.p.) (1)

098.6 °

(Total for Question is 5 marks)

8. The diagram shows a parallelogram.

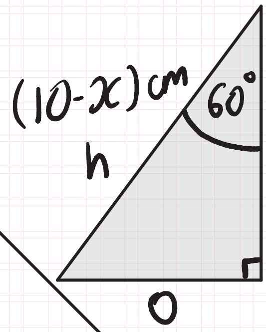
Do this first to work out height of parallelogram



The area of the parallelogram is greater than 15 cm²

(a) Show that $2x^2 - 21x + 40 < 0$

$150 - 90 = 60$



SOHCAHTOA

$\cos\theta = \frac{a}{h}$

$\cos 60° = \frac{a}{10-x}$

$(10-x)\times\frac{1}{2} = \frac{a}{\cancel{10-x}} \times (\cancel{10-x})$

$a = \frac{10-x}{2}$

Area parallelogram = base x height

(1)

$\text{Area} = (2x-1) \times \frac{10-x}{2}$

$= \frac{(2x-1)(10-x)}{2}$

$= \frac{20x - 2x^2 - 10 + x}{2}$

(1) $= \frac{-2x^2 + 21x - 10}{2}$

$\frac{-2x^2 + 21x - 10}{2} > 15$

$-2x^2 + 21x - 10 > 30$

$-2x^2 + 21x - 40 > 0$

$x -1$ $x -1$ (1)

$2x^2 - 21x + 40 < 0$

Flip inequality when multiply by -1

(3)

(b) Find the range of possible values of x.

$2x^2 - 21x + 40 < 0$ 1

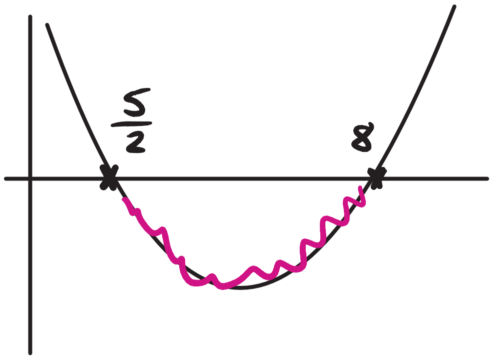
$(2x - 5)(x - 8) < 0$

$(2x-5)(x-8) = 0$

$2x - 5 = 0$ $\quad$ $x - 8 = 0$

$2x = 5$ $\quad$ $x = 8$

$x = \frac{5}{2}$ 1

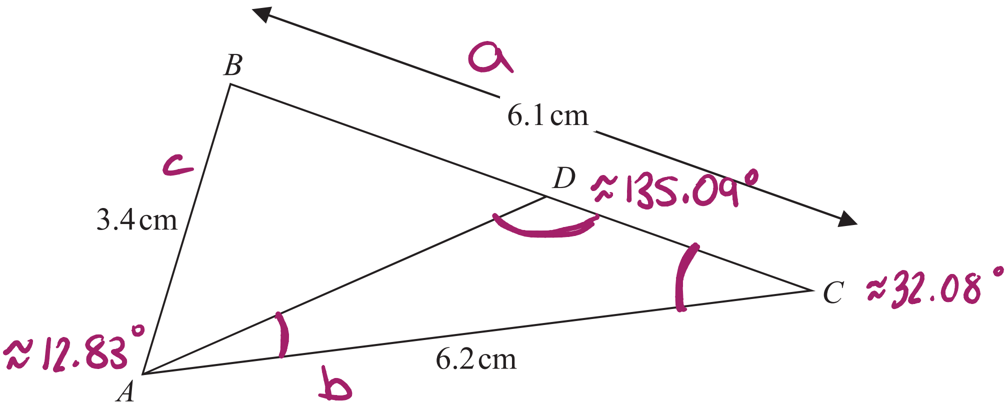


1

$\frac{5}{2} < x < 8$

(3)

9. The diagram shows triangle *ABC*.



$AB = 3.4$ cm $\quad AC = 6.2$ cm $\quad BC = 6.1$ cm

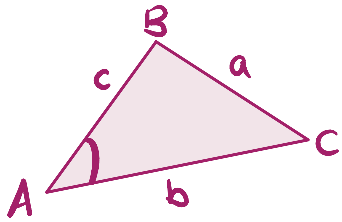
D is the point on *BC* such that

$$\text{size of angle } DAC = \frac{2}{5} \times \text{size of angle } BCA$$

Calculate the length *DC*.
Give your answer correct to 3 significant figures.
You must show all your working.

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

cosine rule



$$c^2 = b^2 + a^2 - 2ba\cos(C)$$

(1) $3.4^2 = 6.2^2 + 6.1^2 - 2(6.2)(6.1)[\cos(C)]$

$11.56 = 75.65 - 75.64\cos(C)$

$+75.64\cos(C) \quad +75.64\cos(C)$

$11.56 + 75.64\cos(C) = 75.65$

$-11.56 \quad -11.56$

$\frac{\cancel{75.64}\cos(C)}{\cancel{75.64}} = \frac{64.09}{75.64}$

$\cos(C) = \frac{64.09}{75.64}$

$C = \cos^{-1}\left(\frac{64.09}{75.64}\right)$

$= 32.08046913°$ (1)

Size angle $DAC = \frac{2}{5} \times 32.080...$

$= 12.83218765°$

Size angle $ADC = 180 - 32.080... - 12.832...$

$= 135.0876432°$

$$\frac{DC}{\sin(DAC)} = \frac{AC}{\sin(ADC)}$$

sine rule (1)

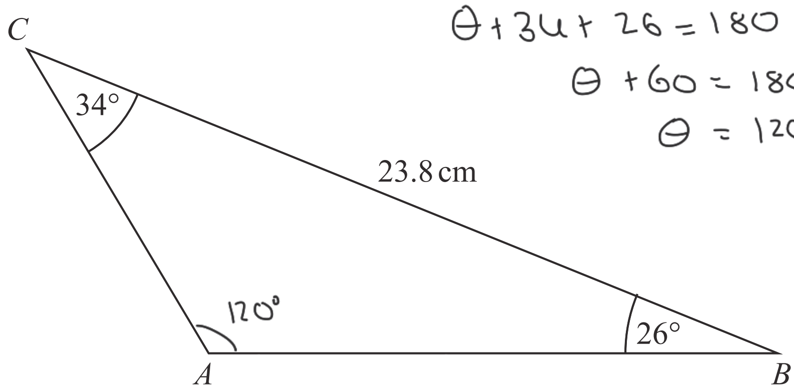
$$\frac{DC}{\sin(12.832...°)} = \frac{6.2}{\sin(135.087...)}$$ (1)

(1) $DC = \frac{6.2}{\sin(135.087...)} \times \sin(12.832...°)$

1.95 cm (3sf) .... cm

$DC = 1.95035... = 1.95$ cm (3sf)

10. Here is triangle *ABC*.



$\theta + 34 + 26 = 180$

$\theta + 60 = 180$

$\theta = 120°$

Work out the length of *AB*.
Give your answer correct to 1 decimal place.

$$\frac{23.8}{\sin 120} = \frac{AB}{\sin 34}$$

$\times \sin 34$ $\times \sin 34$

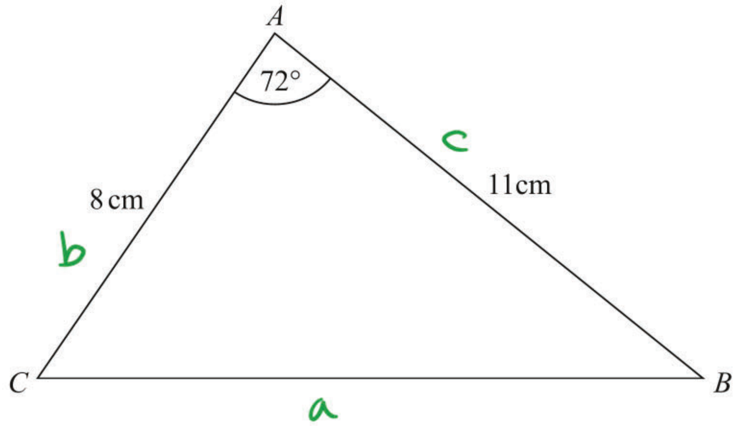
$$AB = \frac{23.8 \times \sin 34}{\sin 120}$$

$$= 15.367\ldots \approx 15.4\text{cm}$$

.... cm

(Total for Question is 3 marks)

11. Here is triangle *ABC*.



(a) Find the length of *BC*.
Give your answer correct to 3 significant figures.

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$a^2 = 8^2 + 11^2 - 2(8)(11)\cos(72°)$ 1

$a^2 = 130.61...$ 1

$a = \sqrt{130.61...} = 11.4286... = 11.4$ (3sf)

1

11.4 cm

(3)

(b) Find the area of triangle *ABC*.
Give your answer correct to 3 significant figures.

area triangle $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}cb\sin A$ 1

$= \frac{1}{2}(11)(8)\sin(72°)$

$= 41.846... = 41.8\text{ cm}^2$ (3sf)

1

41.8 cm²

(2)